

Problem of the Week

Problem B

Making Your Mark

Problem

A group of five students decided to mark up a 100s chart in different ways.

- Zaffar shaded the numbers divisible by 2;
 - Yolanda marked the numbers divisible by 3 with a check mark in the upper right corner;
 - Xerxes circled the numbers divisible by 5;
 - William marked the numbers divisible by 7 with an X in the upper left corner; and
 - Violet underlined the prime numbers.
- a) Did at least one symbol appear on all 100 numbers on the chart?
- b) Which numbers had the most symbols?
- c) How many symbols appeared on the numbers in part b)?
- d) If the chart went beyond 100, what would be the least number that has the symbols of Zaffar, Yolanda, Xerxes, and William?
- e) Is there a number greater than 100 which would satisfy the conditions of all five students? Explain.

Solution

The completed chart is shown following the answers below.

- a) At least one symbol appeared on all numbers except 1.
- b) The numbers which had the greatest number of symbols were 30, 42, 60, 70, 84, and 90.
- c) Each of the numbers in part b) had 3 symbols. The numbers 30, 60, and 90 are divisible by 2, 3, and 5; the numbers 42 and 84 are divisible by 2, 3, and 7; and the number 70 is divisible by 2, 5, and 7.
- d) A number which satisfies the conditions of Zaffar, Yolanda, Xerxes, and William would have to be a multiple of each of 2, 3, 5, and 7. Thus the least such number would be $2 \times 3 \times 5 \times 7 = 210$.





- e) To satisfy the conditions of all five students, a number would have to be both prime AND a multiple of 210. This is a contradiction, since a prime is only divisible by 1 and by itself. Thus no such number exists.

100's Chart

1	<u>2</u>	<u>3</u> [✓]	4	<u>5</u>	6 [✓]	7 ^x	8	9 [✓]	<u>10</u>
<u>11</u>	<u>12</u>	<u>13</u>	14 ^x	<u>15</u> [✓]	16	<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
21 ^x	22	<u>23</u>	<u>24</u>	<u>25</u>	26	27 ^x	28 ^x	<u>29</u>	30 ^x
<u>31</u>	32	<u>33</u>	34	35 ^x	<u>36</u>	<u>37</u>	38	39 ^x	<u>40</u>
<u>41</u>	42 ^x	<u>43</u>	44	<u>45</u> [✓]	46	<u>47</u>	<u>48</u>	49 ^x	<u>50</u>
<u>51</u>	52	<u>53</u>	<u>54</u>	<u>55</u>	56 ^x	57 ^x	58	<u>59</u>	60 ^x
<u>61</u>	62	63 ^x	64	<u>65</u>	<u>66</u>	<u>67</u>	68	69 ^x	70 ^x
<u>71</u>	<u>72</u>	<u>73</u>	74	<u>75</u> [✓]	76	77 ^x	<u>78</u>	<u>79</u>	<u>80</u>
<u>81</u>	82	<u>83</u>	84 ^x	<u>85</u>	86	87 ^x	88	<u>89</u>	90 ^x
91 ^x	92	93 ^x	94	<u>95</u>	<u>96</u>	<u>97</u>	98 ^x	99 ^x	<u>100</u>

